30 YEARS OF EXPERIENCE WITH THE WAVE EQUATION SOLUTION BASED ON THE METHOD OF CHARACTERISTICS

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This paper will describe a brief history of the method of characteristics as an accurate solution of the wave equation, and the author's experiences with the method of characteristics over a period of 30 years.

As a result of the offshore pile driving activities associated with the development of the North Sea oil fields in the 1970's, two Dutch companies initiated extensive research efforts. Heerema started to research the dynamic behavior of soil during driving and HBG (Hollandsche Beton Groep) extended the method of characteristics with a straightforward theoretical solution for the shaft friction along a pile.

The research institute TNO started the development of the wave equation program TNOWAVE in the late 1970's based on HBG's friction model extension for the method of characteristics. Today the program has a worldwide application in the field of pile testing. This paper will describe its development and application over time and its relation to today's applications for driveability studies for impact hammers (PDPWAVE) and vibratory hammers (VDPWAVE), signal matching for Dynamic Load Testing (DLTWAVE), Pile Integrity Testing signal matching (SITWAVE), and Statnamic simulation (STNWAVE).

The paper will highlight that these developments have been the result of multi-disciplinary and multi-national efforts and encourage geotechnical engineers to seek further developments of the wave equation program in a similar manner. Finally, the paper will indicate some areas where such further developments are required.

Introduction

The Dutch author Jaap ter Haar once remarked that "he who lives in the present and doesn't know the past, floats aimlessly and dangerously into the future". In response to that observation Ter Haar spent 5 years in the early 1970's writing a series of four books covering the Dutch history from the prehistory to the then present. Having spent more than 30 years dealing with the Wave Equation Solution based on the Method of Characteristics, the author is regularly reminded how true this observation is. Engineers today take for granted Pile Driving Analysis (PDA) and Signal Matching Techniques to determine pile capacities, without realizing first that these techniques are relatively new, and secondly that they are the outcome of multi-national and multi-disciplinary efforts.

Last year, during the 7th International Conference on the Application of Stress Wave Theory to Piles in Kuala Lumpur, Malaysia a paper was presented covering some of the author's experience with the method of characteristics with the focus on the algorithm used in TNOWAVE. This paper will expand on that topic by looking at the subject of the Wave Equation Solution application in pile foundation technology from a different angle: what can we learn from the developments in the recent past, not only to prevent us from floating aimlessly into the future, but more importantly to make us see more clearly how and in which areas to improve the current technology.

The Pre-History

In the 1860's a Frenchman, A.J.C Barre de Saint Venant, applied the principles of conservation of mass and momentum to the water flow in an open channel. The application resulted in two quasi-linear differential equations, for which de Saint Venant produced a theoretical solution, the so-called method of characteristics. This method was then used for a variety of hydrology related issues, such as to predict the propagation of tidal waves, based on the work of J. Massau and J.C.Schonfield.

Starting in the 1930's Saint Venant's equations were applied to pile driving around the world, such as by L.H. Donnell in the United States, D.V. Isaacs in Australia, and W.H. Glanville in England. In 1938 the latter directed the first comprehensive study aimed at understanding cracking in concrete piles at both the top and the bottom during pile driving. As part of this work, measurements were taken during pile driving using what was considered at that time portable equipment in a construction trailer. As such Glanville is truly the pioneer in the field of PDA, as it would take some 18 years before similar work was done in the Netherlands (by A. Verduin in 1956) and some 25 years before it was done here in the United States (by G.G. Goble in 1964).

Just as Glanville can be considered the father of PDA, Isaacs can be considered the first to ever use wave equations for modeling pile driving. But apart from that, he should also be remembered for his insights on the issue of safety factors. In an article published in 1931 in the Journal of the Institution of Engineers Australia, Isaacs wrote the following on this subject:

> *It should be remembered, however, that these are not true factors of safety, but include a "factor of ignorance." The author suggests that when the ultimate resistance of any pile has been determined, in fixing the factor of safety...the most unfavorable conditions possible in the supporting strata should be judged (the range of conditions possible being narrowed with better knowledge of the subsurface conditions and of the possibility of disturbance from extraneous sources) and a proportion of the factor of safety -- a "factor of ignorance" -- then allowed in respect to these possible conditions, the manner of determining the ultimate load, and the type of loading to be borne. The remaining proportion of the factor of safety -- or true margin of safety -- should be approximately constant for all classes of loading and foundation conditions involving the same value of loss in case of failure; and the overall factor of safety...will then be equal to the product of the true factor of safety with the "factor of ignorance." (p. 305)*

This way of dealing with safety factors was and remains extremely insightful, and is just another example why it is important to know the past.

The original method of characteristics was valid for a free rod, not interacting with shaft friction or toe resistance. After all, the method is based on the phenomenon that stress waves propagate unaltered with a characteristic stress wave velocity. In 1956 Dutch scientist G. de Josseling de Jong proposed a model to incorporate the toe resistance into the method of characteristics (Fig. 1), and then in 1974 the Dutch company HBG extended the method by formulating the theoretical solution for piles with shaft friction. This development was part of HBG's development of the Hydroblok impact hammer, a high-tech hammer using nitrogen cushioning to improve efficiency.

Until 1974 when friction was introduced, the partial differential equation could only be solved analytically if the friction was represented as an analytical function. The solution was then found by integral transforms (such as the Laplace transform) and obtained in the form of Fourier series (Van Koten, et al. 1980). However, if the shaft friction was assumed to depend on the velocity or displacement, there was no theoretical solution and a numerical integration of the differential equation had to be used, as was done at that time in the numerical wave equation program developed by E.A.L. Smith. For this purpose the pile was modeled by a number of point masses, with the shaft friction and toe resistance introduced through a series of springs with dash-pots connected to the point masses (the so-called Lumped Model, Fig. 2).

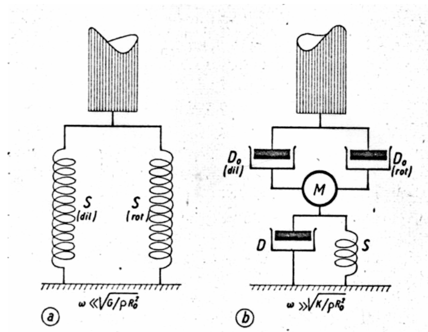

Fig. 1. Toe resistance model for method of characteristics for low and high frequencies proposed by De Josseling de Jong (1956)

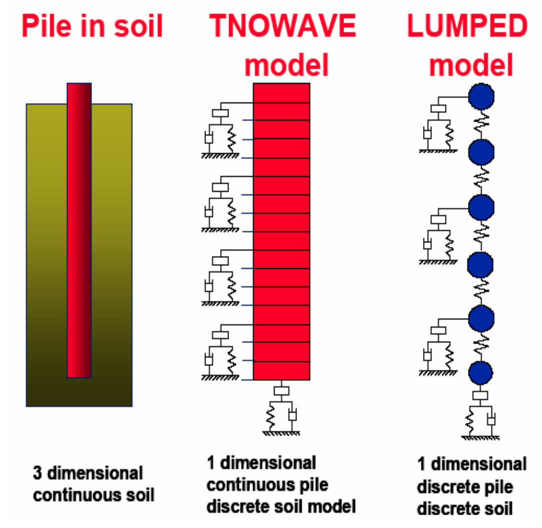

Fig. 2. Wave Equation Models

To be able to generate a theoretical solution, the HBG took a different approach to account for the friction, which – with hindsight – was really very simple: "when the friction is concentrated at a number of points, the parts of the pile between these points are not subject to friction and the simple stress wave theory is valid for them", and the discontinuities that occur at the points where the friction is modeled can be easily dealt with. This method formed the basis for HBG's computer program PILEWAVE that was released in 1974, a few years before the WEAP program was released in the USA and TNOWAVE was released in The Netherlands.

TNOWAVE

Although it is a small country, The Netherlands has a huge piling market because of the poor soil conditions. Every year between 700,000 and 1,000,000 pre-cast piles are driven and a similar number of cast in situ piles are installed. It is therefore not surprising that there was a need for basic research in the area of pile foundations. Consequently in the 1960's a group was established within the Building and Construction Research Department of the Dutch organization for Applied Scientific Research (TNO), the second largest R&D organization in Europe. The Foundation Pile Diagnostic Systems (FPDS) Group, as it was initially called before it was renamed Profound, took on a broad range of activities for its clients in foundation pile testing with state-of-the-art technology that was developed in research and development projects.

In the 1970's this group started to really focus on the field of stress wave applications when two Dutch companies (Heerema and HBG) together with TNO intensified their research in this area because of the offshore pile driving activities associated with the development of the North Sea oil fields.

In 1975 Heerema performed research at the TNO laboratories to investigate soil behavior during pile driving to develop models for pile driveability analysis, elaborating on the work done by H.M. Coyle and G.C. Gibson in the United States. At the same time HBG used the stress-wave application in their development of the Hydroblok, which lead to the release of PILEWAVE in 1974.

Following the release of PILEWAVE, TNO wanted to develop its own wave equation program. While it was considered to use Smith's algorithm as the basis for such a program, it was decided to use the method of characteristics instead, and the first issue of the program TNOWAVE was released in 1978. In the following years much work was done to better understand the dynamic soil model parameters and to establish signal matching techniques, similar to CAPWAP. The first commercial version of the DLTWAVE module was released in 1982.

In the second half of the 1980's TNOWAVE was extended to vibratory pile driving prediction, again in response to developments in the offshore oil & gas industry. At that time the theories to predict the performance of a vibro-hammer were based on a single, lumped-mass pile model. This approach is valid for relatively short piles, but not for long piles (such as used offshore). A long pile is not moving as a single lumped mass, and stress wave phenomena have to be taken into account. Using the method of characteristics the VDPWAVE module was developed, and following its release in 1988 it was used to demonstrate that long offshore piles could be installed with vibro-hammers (Jonker & Middendorp, 1988).

From the above it is obvious that the early development of TNOWAVE was heavily affected by the offshore oil & gas industry (just as the

United States Federal Highway Administration and the New York Department of Transportation had a strong influence on the development of the WEAP program in the United States). By the late 1980's and early 1990's there were several features added to the program to address needs in the pile foundation industry in general.

First, the Dutch pre-cast piling industry wanted to know the minimum amount of steel required to allow pile driving without loss of quality and reliability. Since the main function of the reinforcement is to prevent or at least reduce cracking during pile driving, a numerical crack model was developed and implemented into TNOWAVE in 1988. This model contains the following options:

- Simultaneous wave propagation in the concrete and the reinforcement
- Bond forces between concrete and reinforcement
- Pre-stressing
- Cracking models
- Multiple cracking
- Opening and closing of cracks

Secondly, around that same time the TNOWAVE signal matching technology was extended to pile integrity testing, and in 1988 the SITWAVE module was released. Later that year the module was successfully tested on piles with known defects.

Finally, in the early 1990's the program was expanded one more time with the release of the STNWAVE module to cover the development of Statnamic testing, which was developed by TNO and Berminghammer. It is interesting to note that this module strictly speaking doesn't belong in TNOWAVE. Given the long duration of the Statnamic test (compared to a dynamic load test) it was first assumed and later demonstrated that the pile could be modeled as a lumped mass with springs and that stress wave phenomena did not have to be taken into account.

The Present and the Future

It is clear that the stress-wave programs, such as TNOWAVE and WEAP, have allowed and continue to allow lowering of the pile foundation costs. First, through the simulation mode it provides engineers the opportunity to optimize the design of both the pile itself and the pile foundation. Secondly, through the signal matching mode it allows the widespread use of dynamic load testing as well as Statnamic testing, which provide valuable information on the pile bearing capacity and therefore enable even further optimization of the pile foundation design.

With the full implementation in 2007 of the Load and Resistance Factor Design (LRFD) methodology as mandated by the Federal Highway Administration, pile testing will become more widespread, and stress-wave program applications are therefore likely to increase in the coming years.

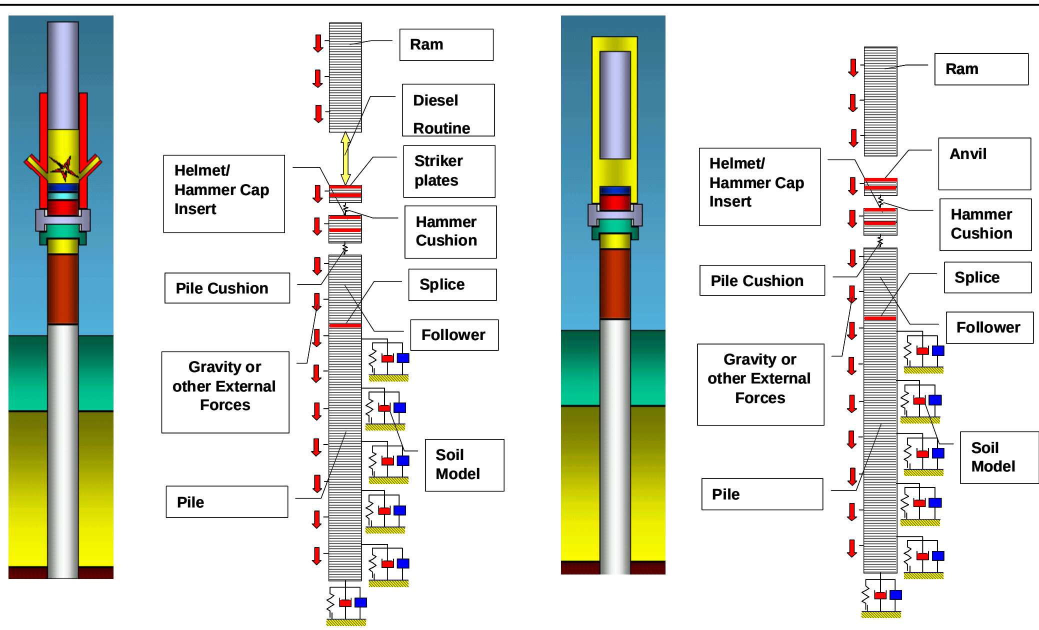

Fig 3. TNOWAVE models for Diesel and Hydraulic Hammers

Another current application of the stress-wave programs is the optimization of hammers and pile cushions. Profound has developed a close working relationship with several hammer manufacturers around the world (such as IHC in the Netherlands, APE in the United States and Berminghammer in Canada). These relationships provide Profound with useful suggestions to upgrade the TNOWAVE program to meet the requirements of the hammer manufacturers, and they are likely to be at least one of the driving forces behind the future developments of the programs.

A similarly obvious development will be the increased use of wireless data transmission. This will reduce and eventually eliminate the need for cables to connect the sensors mounted on the piles to the workstations, and also allow remote monitoring with data automatically forwarded from the construction site to the design office using cell phones.

Along the same lines, the need for dedicated work stations is likely to be eliminated by the use of PDA and other pile testing options that can be plugged into notebook computers by simple USB connections.

But apart from that, there will be other developments. Some of these will be considered (at least with hindsight) so obvious that future engineers will wonder why it took so long before such developments took place, just as it is now blatantly obvious how pile friction should be dealt with in the stress-wave program.

One area that can never be overlooked is soil modeling. While the TNOWAVE soils investigation module handles many different types of soil investigations (e.g. CPT, SPT, DMT and PMT or bore hole results), this part is undoubtedly the weakest link in the overall hammer-pile-soil model. To borrow a phrase from Isaacs, the *factor of ignorance* is the greatest in the soil modeling, and therefore research efforts should be directed towards this area, making full use of the ever increasing computing capabilities. In this regard the efforts in the Netherlands surrounding the Geotechnical Exchange Format (GEF) should be noted. Soil investigation data (like CPT results) are recorded in a standardized digital format (GEF) and the TNOWAVE program can than read these GEF files, after which the soil investigation module automatically determines the soil types and then converts the soil investigation data into the fundamental static and dynamic soil parameters used in the algorithms.

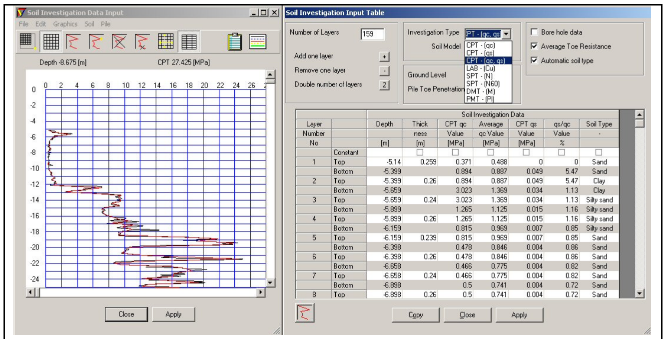

Fig. 4. Soil investigation input module

A second focus area should be the application of the stress wave program to pile driving with vibratory hammers. While this is a routine operation, there is a lack of measurement data documenting the operation. So in order to bring VDPWAVE to the next level there is a need not only to collect data during the vibratory driving process, but also to analyze this information and compare it with the computer predictions.

As engineers we need to keep pushing the envelope for these developments to take place, but at the same time we need to keep a close eye at other disciplines. We need to remember that the basic equations used in the stress-wave programs did not come from geotechnology. In other words, the next big development may already be out there, waiting for a geotechnical engineer to apply it to pile foundations.

Similarly we need to be open to developments elsewhere in the world. In this regard the international efforts of DFI are to be applauded and supported. Many engineers work for companies with international operations, which provides ample opportunity for interaction with engineers of other nationalities. The internet allows easy communication with people around the world, either through e-mail or message boards such as Geoforum. Once again, the problem at hand may not only have been encountered by another engineer, but the solution may already have been developed.

After all, the pile driving application of the wave equation solution based on the methods of characteristics is the result of multi-national and multi-disciplinary efforts, and any substantial developments are more than likely to come from similar efforts like that rather than from a narrowly focused effort.

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